Black Hole Entropy from Horizon Conformal Field Theory

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String theory and "quantum geometry" have recently offered independent statistical mechanical explanations of black hole thermodynamics. But these successes raise a new problem: why should models with such different microscopic degrees of freedom yield identical results? I propose that the asymptotic behavior of the density of states at a black hole horizon may be determined by an underlying symmetry inherited from classical general relativity, independent of the details of quantum gravity. I offer evidence that a two-dimensional conformal symmetry at the horizon, with a classical central extension, may provide the needed behavior.

1. SOME QUESTIONS

More than 25 years have now passed since the discovery by Bekenstein [1] and Hawking [2] that black holes are thermal objects, with characteristic temperatures and entropies. But while black hole thermodynamics is by now well established, the underlying statistical mechanical explanation remains profoundly mysterious. Recent partial successes in string theory [3] and the "quantum geometry" program [4] have only added to the problem: we now have several competing microscopic pictures of the same phenomena, with no clear way to understand why they give identical results.

The mystery is deepened when we recall that the original analysis of Bekenstein and Hawking needed none of the details of quantum gravity, relying only on semiclassical results that had no obvious connection with microscopic degrees of freedom. This is the problem of "universality": why should such profoundly different approaches all give the same answers for black hole temperature and entropy?

I will orient this presentation around a few fundamental questions. At first sight, some of these questions—although not their answers—are obvious, while others may seem more obscure. I hope to show that these are "right" questions, in that they lead toward a plausible solution to the problem of universality in black hole thermodynamics. The solution I suggest is certainly not proven, however, and perhaps at this stage the questions are as important as the answers.

The questions are these:

- Why do black holes have entropy?
- Can black hole horizons be treated as boundaries?
- Why do different approaches to quantum gravity yield the same black hole entropy?
- Can classical symmetries control the density of quantum states?
- Can two-dimensional conformal field theory be relevant to realistic (3+1)-dimensional gravity?

1.1. Why do black holes have entropy?

Our starting point is the Bekenstein-Hawking entropy

$$S = \frac{A}{4\hbar G} \tag{1}$$

for a black hole of horizon area A. It is possible, of course, that black holes are fundamentally unlike any other thermodynamic systems, and that black hole entropy is unrelated to any microscopic degrees of freedom. But if we reject such a radical proposal, then even knowing nothing about quantum gravity, we can make some reasonable guesses.

First, the underlying microscopic degrees of freedom must be quantum mechanical, since S

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depends on Planck's constant. Second, they must be, in some sense, gravitational, since S depends on Newton's constant. It is thus reasonable to suppose that they are quantum gravitational, though this conclusion is not quite necessary—the relevant degrees of freedom could conceivably be those of a quantum field theory in a classical gravitational background. Third, the dependence of S on the horizon area suggests (though again does not prove) that the degrees of freedom responsible for black hole entropy live on or very near the horizon.

At the same time, we know that the relevant degrees of freedom cannot be the ordinary "graviton" degrees of freedom one expects in quantum gravity. As Bañados, Teitelboim, and Zanelli showed [5], black holes exist in (2+1)-dimensional general relativity, and exhibit the usual thermodynamic behavior. But in 2+1 dimensions there are no gravitons, and the ordinary "bulk" degrees of freedom are finite in number [6]. Indeed, on a spatially compact (2+1)-dimensional manifold, the bulk gravitational degrees of freedom are far too few to account for black hole entropy, and we can obtain the Bekenstein-Hawking entropy from quantum gravity only if we admit extra "boundary" degrees of freedom [7]. Such boundary excitations appear naturally in the Chern-Simons formulation of (2+1)-dimensional gravity as "would-be gauge degrees of freedom," that is, excitations that would normally be unphysical, pure gauge configurations but that become physical in the presence of a boundary [8,9].

In the (2+1)-dimensional theory, there are two obvious candidates for a "boundary," spatial infinity and the black hole horizon. The degrees of freedom at spatial infinity are naturally described by a Liouville theory [10], and it is not obvious that there are enough of them to account for black hole entropy. However, an elegant conformal field theory argument due to Strominger [11] leads to the correct Bekenstein-Hawking formula. Unfortunately, though, the theory at infinity cannot distinguish between a black hole and, for example, a star of the same mass, and cannot easily attribute separate entropies to distinct horizons in multi-black hole spacetimes. For that, we need a "boundary" associated with each horizon. This leads to the next question:

1.2. Can black hole horizons be treated as boundaries?

A black hole horizon is not, of course, a true physical boundary. A freely falling observer can cross a horizon without seeing anything special happen; she certainly doesn't fall off the edge of the universe. To understand the role of a horizon as a boundary, one must think more carefully about the meaning of a black hole in quantum gravity.

Suppose we wish to ask a question about a black hole: for example, what is the spectrum of Hawking radiation? In semiclassical gravity, this is easy—we merely choose a black hole metric as a background and do quantum field theoretical calculations in that fixed curved spacetime.

In full quantum gravity, though, life is not so simple. The metric is now a quantum field, and the uncertainty principle forbids us from fixing its value. We can at most fix "half the degrees of freedom" of the geometry. How, then, do we know whether there is a black hole?

The answer is that we can restrict the metric to one in which a horizon is present. A horizon is a codimension one hypersurface, and we need only "half the degrees of freedom" to fix its properties. This does not, of course, make the horizon a physical boundary, but it makes it a place at which "boundary conditions" are set. In a path integral formulation, for instance, we can impose the existence of a horizon by splitting the manifold into two pieces along some hypersurface and performing separate path integrals over the fields on each piece, with fields restricted at the "boundary" by the requirement that it be a horizon. This kind of split path integral has been studied in detail in 2+1 dimensions [12], where it can be shown that the usual boundary degrees of freedom appear. It seems at least plausible that the same should be true in higher dimensions.

(I should confess that I have been sweeping a rather difficult problem under the rug. It is not clear what kind of horizon one should choose, or what the appropriate boundary conditions should be. Recent work by Ashtekar and collaborators on "isolated horizons" is probably relevant [13], but these results have not yet been applied to this question.)

1.3. Why do different approaches to quantum gravity yield the same entropy?

We next turn to the problem posed at the beginning of this work, that of universality. Ten years ago, if someone had asked for a statistical mechanical explanation of black hole entropy, the best answer would have been, "We don't know." Today we suffer an embarrassment of riches—we have several explanations from string and D-brane theory [3], another from the "quantum geometry" program [4], yet another from Sakharov-style induced gravity [14]. None of these is yet completely satisfactory, but all give the right functional dependence and the right order of magnitude for the entropy. And all agree with the original semiclassical results that were obtained without any assumptions about quantum gravitational microstates.

In one sense, this agreement is not surprising: any quantum theory of gravity had better give back the semiclassical results in an appropriate limit. But the quantity we are investigating, the entropy, is a measure of the density of states, about as quantum mechanical a quantity as one could hope to find. Merely pointing to the semiclassical results does not explain *why* the density of states behaves as it does.

This problem has not yet been solved. But perhaps the most plausible direction in which to look for a solution is suggested by Strominger's recent work [11]. Regardless of the details of the degrees of freedom, any quantum theory of gravity will inherit from classical general relativity a symmetry group, the group of diffeomorphisms. While the commutators may receive quantum corrections of order $\hbar$, we expect the fundamental structure to remain. So perhaps the classical structure of the group of diffeomorphisms is sufficient to govern the gross behavior of the density of quantum states.

1.4. Can classical symmetries control the density of quantum states?

Symmetries determine many properties of a quantum theory, but one does not ordinarily think of the density of states as being one of these properties. In one large set of examples, however, the two-dimensional conformal field theories, the symmetry group does precisely that.

Consider a conformal field theory on the complex plane. The fundamental symmetries are the holomorphic diffeomorphisms $z \to z + \epsilon f(z)$. If one takes a basis $f_n(z) = z^n$ of holomorphic functions and considers the corresponding algebra of generators L_n of diffeomorphisms, it is easy to show that [15]

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n} \tag{2}$$

where the constant c, the central charge, is a measure of the conformal anomaly. The algebra (2) is known as the Virasoro algebra.

As Cardy first showed [16], the central charge c is nearly enough to determine the asymptotic behavior of the density of states. Let Δ_0 be the lowest eigenvalue of the Virasoro generator L_0, that is, the "energy" of the ground state, and denote by $\rho(\Delta)$ the density of eigenstates of L_0 with eigenvalue Δ. Then for large Δ,

$$\rho(\Delta) \sim \exp\left\{2\pi\sqrt{\frac{c_{\text{eff}}\Delta}{6}}\right\} \tag{3}$$

where

$$c_{\text{eff}} = c - 24\Delta_0. \tag{4}$$

A careful derivation of the Cardy formula (3)–(4) is given in reference [17]. The proof involves a simple use of duality:

Consider a conformal field theory on a cylinder; then analytically continue to imaginary time, and compactify time to form a torus. In addition to the familiar "small" diffeomorphisms, the torus admits "large" diffeomorphisms, one of which is an exchange of the two circumferences. Under such an exchange, the angular and time coordinates are swapped. Since the theory is conformal, we can always rescale to normalize the angular circumference to have length 1. The exchange of circumferences is then a map from t to $1/t$, or equivalently from energy E to $1/E$. The high energy states are thus connected by symmetry to low energy states, and one can obtain their gross properties—in particular, the behavior of the density of high-E states—from knowledge of low-E states. The dependence of the Cardy formula on the central charge appears because the theory is not really quite scale-invariant: c is a conformal anomaly, and the rescaling of the angular circumference introduces factors of c.

The central charge of a conformal field theory ordinarily arises from operator ordering in the quantization. But as Brown and Henneaux have stressed [18], a central charge can appear *classically* for a theory on a manifold with boundary. In the presence of a boundary, the generators of diffeomorphisms typically acquire extra "surface" terms, which are required in order that functional derivatives and Poisson brackets be well-defined [19]. These surface terms are determined only up to the addition of constants, but the constants may not be completely removable; instead, they can appear as central terms in the Poisson algebra of the generators.

The canonical example of such a classical central charge is (2+1)-dimensional gravity with a negative cosmological constant $\Lambda = -1/\ell^2$ [18]. For configurations that are asymptotically anti-de Sitter, the algebra of diffeomorphisms acquires a surface term at spatial infinity, and the induced algebra at the boundary becomes a pair of Virasoro algebras with central charges

$$c = \bar{c} = 3\ell/2G. \tag{5}$$

Strominger's key observation [11] was that if one takes the eigenvalues of L_0 and $\bar{L}_0$ that correspond to a black hole,

$$M = (L_0 + \bar{L}_0)/\ell, \quad J = L_0 - \bar{L}_0 \tag{6}$$

and assumes $\Delta_0 = 0$, the Cardy formula (3)–(4) gives the standard Bekenstein-Hawking entropy (1) for the (2+1)-dimensional black hole.

Strominger's derivation is an elegant "existence proof" for the idea that black hole entropy can be determined by classical symmetries. As a general argument, though, it has two obvious limitations. First, it uses features peculiar to 2+1 dimensions. In particular, the relevant Virasoro algebras have a natural geometrical meaning: they are the symmetries of the two-dimensional boundary of three-dimensional adS space. While many of the black holes in string theory have near-horizon geometries that resemble the (2+1)-dimensional black hole [17], others do not, so this limitation is a serious one.

Second, since these Virasoro algebras appear at spatial infinity, they cannot in themselves detect the details of the interior geometry. For example, multi-black hole solutions ought to have a distinct entropy attributed to each horizon, but an asymptotic algebra at infinity can only determine the total entropy. Indeed, the classical central charge at infinity cannot tell whether the configuration is a black hole or a star.

We are thus led to look for generalizations of Strominger's approach to higher dimensions and to boundary terms at individual black hole horizons. It seems natural to start by looking for higher-dimensional generalizations of the Cardy formula. Unfortunately, no such generalizations are known, and the derivation of equations (3)–(4) does not naturally extend to more than two dimensions. We are thus led to our next question:

1.5. Can two-dimensional conformal field theory be relevant to realistic (3+1)-dimensional gravity?

We have one example in which symmetries can control the density of quantum states: conformal field theory in two dimensions. For this fact to be relevant to realistic (3+1)-dimensional gravity, we must argue that some two-dimensional submanifold of spacetime plays a special role in black hole thermodynamics. This is in fact the case: in the semiclassical approach to black hole thermodynamics in any dimension, all of the interesting physics takes place in the "r–t plane."

Let us consider for simplicity an n-dimensional Schwarzschild black hole, analytically continued to "Euclidean" signature. (The generalization to more complicated black holes is fairly straightforward.) Near the horizon, the metric takes the form

$$ds^2 \approx \frac{r_+}{r - r_+} dr^2 + \frac{r - r_+}{r_+} d\tau^2 + r_+^2 d\Omega^2 \tag{7}$$

where the horizon is located at $r = r_+$. It is well known that the Hawking temperature can be obtained by demanding that there be no conical singularity in the r–τ plane. Indeed, if we transform to new coordinates

$$R = 2\sqrt{r_+(r-r_+)}, \quad \hat{\tau} = \frac{\tau}{2r_+} \tag{8}$$

the metric (7) becomes

$$ds^2 \approx dR^2 + R^2 d\hat{\tau}^2 + r_+^2 d\Omega^2. \tag{9}$$

Smoothness at $R = 0$ then requires that $\hat{\tau}$ be an ordinary angular coordinate with period 2π, i.e., that τ have a period $\beta = 4\pi r_+$, the correct inverse Hawking temperature. Note that this argument depended only on the geometry of the r–τ plane near the horizon, and not on the angular coordinates.

What is less well known is that the semiclassical computation of the entropy also depends only on the near-horizon geometry of the r–τ plane. If one chooses boundary conditions appropriate for the microcanonical ensemble, the classical action of general relativity reduces to the Einstein-Hilbert action for a "cylinder" $\Delta_\epsilon \times S^{n-2}$, where Δ_ϵ is a disk of radius ϵ around the point $r = r_+$ in the r–τ plane [20]. It may be shown that the action factorizes, becoming

$$\lim_{\epsilon\to 0} I = \frac{1}{8\pi G}\chi(\Delta_\epsilon) \times \mathrm{Vol}(S^{n-2}) \tag{10}$$

where $\chi(\Delta_\epsilon)$ is the Euler characteristic of Δ_ϵ.

In the standard Euclidean path integral approach to black hole thermodynamics, the action $I/\hbar$, evaluated with appropriate boundary conditions, gives the leading order contribution to the entropy. It is evident from equation (10) that the "transverse" factor $\mathrm{Vol}(S^{n-2})$ is needed to obtain the correct entropy—it gives the factor of area in equation (1). But this term is also nondynamical, merely providing a fixed multiplicative factor; it is the topology of the r–τ plane that distinguishes the black hole configuration from any other.

To better understand the relevant conformal field theory, it is useful to combine the coordinates R and $\hat{\tau}$ in equation (9) into a complex coordinate $z = Re^{i\hat{\tau}}$:

$$z \sim \exp\left\{\frac{1}{2r_+}\left[i\tau + r_+ \ln\left(\frac{r}{r_+} - 1\right)\right]\right\}. \tag{11}$$

Continuing back to Lorentzian signature, we see, perhaps not surprisingly, that "holomorphic" and "antiholomorphic" functions correspond to functions of $t \pm r_*$, where r_* is the usual "tortoise coordinate."

These results suggest two possible strategies for further investigating the statistical mechanics of black hole entropy. We can try to dimensionally reduce general relativity to two dimensions in the vicinity of a black hole horizon, and see whether we can identify a conformal field theory and determine the central charge. Alternatively, we can look at the Poisson algebra of the generators of diffeomorphism and see whether an appropriate subgroup of transformations of the r–t plane acquires a central charge.

The first of these strategies has been explored by Solodukhin [21], who has shown that the near-horizon dimensional reduction of general relativity leads to a Liouville theory with a calculable central charge. While there is still some uncertainty about the proper choice of the eigenvalue Δ in the Cardy formula, it seems likely that the symmetries yield the correct Bekenstein-Hawking entropy. The second strategy is my next topic.

2. ENTROPY IN ANY DIMENSION

The arguments of the preceding section have led us to a possible strategy for understanding the universal nature of the Bekenstein-Hawking entropy. We begin with classical general relativity on a manifold with boundary, imposing boundary conditions to ensure that the boundary is a black hole horizon. We next investigate the classical Poisson algebra of the generators of diffeomorphisms on this manifold, concentrating in particular on the subalgebra of diffeomorphisms of the r–t plane. We expect this subalgebra to acquire a classical central extension, with a computable central charge and with some eigenvalue Δ of L_0 associated with the black hole. We then see whether these values of c and Δ yield, via the Cardy formula, the correct asymptotic behavior of the density of states. If they do, we will have demonstrated that the Bekenstein-Hawking entropy is indeed governed by symmetries, independent of the finer details of quantum gravity.

The exploration of this strategy was begun in references [22] and [23]. While conclusive answers have not yet been obtained, the results so far suggest that the program may succeed. I will now briefly summarize the progress.

2.1. Central terms

In classical general relativity, the "gauge transformations" have generators $H[\xi]$ that can be written as integrals of the canonical constraints over a spacelike hypersurface. For a spatially closed manifold, these generators obey the standard Poisson algebra $\{H[\xi_1], H[\xi_2]\} = H[\{\xi_1 \xi_2\}]$, where the bracket $\{\xi_1 \xi_2\}$ is the usual Lie bracket for vector fields, or the closely related "surface deformation" bracket of the canonical algebra [18]. As DeWitt [24] and Regge and Teitelboim [19] noted, however, the presence of a boundary can alter the generators $H[\xi]$: in order for their functional derivatives and Poisson brackets to be well-defined, one must add surface terms $J[\xi]$, whose exact form depends on the choice of boundary conditions.

The presence of such surface terms can, in turn, affect the algebra of constraints. If one writes $H[\xi]+J[\xi] = L[\xi]$, one generically finds an algebra of the form

$$\{L[\xi_1], L[\xi_2]\} = L[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2] \tag{12}$$

where the central term $K[\xi_1, \xi_2]$ depends on the metric only through its fixed boundary values. As Brown and Henneaux pointed out [18], one can evaluate this central extension by looking at the surface terms in the variation $\delta_{\xi_1} H[\xi_2]$, where δ_{ξ_1} means the variation corresponding to a diffeomorphism parametrized by the vector ξ_1. Indeed, since such a variation is generated by $L[\xi_1]$, it is clearly related to the brackets (12). A general expression for these brackets, derived using Wald's covariant "Noether charge" formalism [25], is given in reference [23].

2.2. Boundary conditions

To apply these techniques to a black hole, we must next decide how to impose boundary conditions that imply the presence of a horizon. This is perhaps the most delicate aspect of the program, and it is not yet fully understood.

One possible procedure [22] is to look at the functional form of the metric of a genuine black hole in some chosen coordinate system near its horizon, and to restrict oneself to metrics that approach this form near the boundary. This seems fairly straightforward, but it may be too coordinate-dependent, and it is not obvious how fast one should require metric components to approach their boundary values. A second procedure [23] is to impose the existence of a local Killing horizon in the vicinity of the boundary. This has the advantage of covariance, but it is probably too restrictive for a dynamical black hole, and it is again unclear how fast the geometry should approach the desired boundary values.

Each of these choices of boundary conditions leads to the constraint algebra (12), with a nonvanishing central term $K[\xi_1, \xi_2]$. Moreover, when restricted to diffeomorphisms of the r–t plane, this algebra reduces to something that is almost a Virasoro algebra. In the covariant phase space approach of reference [23], for example, one finds a central term of the form

$$K[\xi_1, \xi_2] \sim \int_{\mathcal{H}} \hat{\epsilon} \left(D\xi_1 D^2 \xi_2 - D\xi_2 D^2 \xi_1 \right) \tag{13}$$

where $\mathcal{H}$ is a constant-time cross section of the horizon, $\hat{\epsilon}$ is the induced volume element on $\mathcal{H}$, and $D = \partial/\partial v$ is a "time" derivative, that is, a derivative along the orbit of the Killing vector at the horizon.

If this expression included an integration over the time parameter v, equation (13) would be precisely the ordinary central term for a Virasoro algebra, and Fourier decomposition would reproduce equation (2). Unfortunately, no such v integration is present. This mismatch of integrations was first noted by Cadoni and Mignemi [26] in the study of symmetry algebras in (1+1)-dimensional asymptotically anti-de Sitter spacetimes. They suggested that one could define new "time-averaged" generators, which would then obey the usual Virasoro algebra. Alternatively, in more than 1+1 dimensions one can add extra "angular" dependence to the r–t diffeomorphisms in such a way that a conventional Virasoro algebra is recovered. It was argued in reference [23] that such angular dependence may be needed to have a well-defined Hamiltonian. However, there is clearly more to be understood.

2.3. Entropy and the Cardy formula

Let us set aside this problem for the moment, and suppose that we have included a suitable angular dependence or an extra integration over v in order to obtain a standard Virasoro algebra. The resulting central charge can then be computed as in reference [18], and the application of the Cardy formula (3)–(4) is straightforward. One finds that both the boundary conditions of reference [22] and those of reference [23] yield the standard Bekenstein-Hawking entropy (1).

Although this analysis was carried out for ordinary black holes in general relativity, it can be easily extended to a number of other interesting problems. The same type of argument yields the correct entropy for cosmological horizons [27], and probably for "Misner strings" in Taub-NUT and Taub-bolt spaces [23]. The analysis can be easily applied to dilaton gravity as well, where it again gives the correct entropy.

Several problems remain. The first two, which are probably related, are to find the right general boundary conditions and to understand the extra integration required in equation (13).

The third is conceptually more difficult. The approach to black hole entropy described here is nondynamical: by imposing boundary conditions at a horizon, we are fixing the characteristics of that horizon, effectively forbidding such processes as Hawking radiation. This is not a terrible thing if we are only interested in equilibrium thermodynamics; the Bekenstein-Hawking entropy, for example, is presumably "really" the entropy of a black hole in equilibrium with its Hawking radiation. To fully understand the quantum dynamics of a black hole, however, one needs a more general approach, in which boundary conditions are strong enough to determine that a black hole is present, but flexible enough to allow that black hole to evolve with time. I leave this as a challenge for the future.

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